Advanced Geometry

Types of polygons

Sides	Name	Internal Degrees	External Degrees
3	Triangle	180	360
4	Quadrilateral	360	360
5	Pentagon	540	360
6	Hexagon	(Add 180 for each side)	360
7	Heptagon		
8	Octagon		
9	Nonagon		
10	Decagon		
n	n-gon	$S = 180(n - 2)$	360

Problem 1: How large is each angle of a regular octagon?

Problem 2: What kind of regular polygon has internal angles of 144º?

Types of triangles

Equal angles open to equal sides
A larger angle opens to a longer side

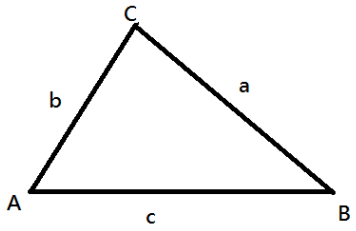
All sides / angles different = Scalene
At least two sides / angles same = Isosceles
All three sides / angles equal = Equilateral or regular

Largest angle	Type
< 90	Acute
= 90	Right
> 90	Obtuse

Problem 3: What is the special name for an isosceles right triangle?

The triangle inequality

"The shortest distance between two points is a straight line"



$$a \le b + c$$
$$b \le a + c$$
$$c \le a + b$$

If we put two of these formulae together (don't sweat the details) we get the nice sandwich inequality below.

$$|a - b| \le c \le a + b$$

The "or equal to" can only be true if the points are allowed to fall on a straight line.

Problem 4: Two sides of a triangle are 3 and 4. How long could the other side be?

[A] 1 [B] 4 [C] 5 [D] 7 [E] 8

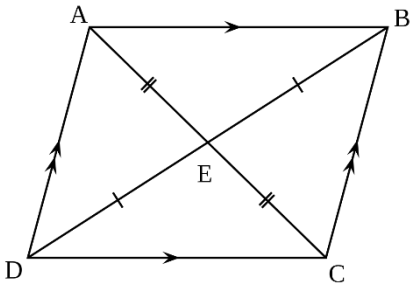
Problem 5: Johnny lives 4 miles from work and 9 miles from college. How far could it be between work and college?

[A] 5 [B] 9 [C] 13 [D] 15

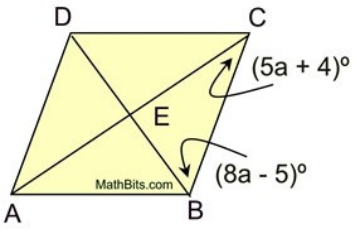
Types of quadrilaterals

Parallelograms

- Two pairs of equal, parallel sides
- Diagonals bisect each other
- Right angles = Rectangle
  - Diagonals are equal
- Equal sides = Rhombus ("Diamond")
  - Diagonals are perpendicular
  - Alt $A = \frac{1}{2} d_1 d_2$ (diagonals)
  - Square = Rectangular rhombus

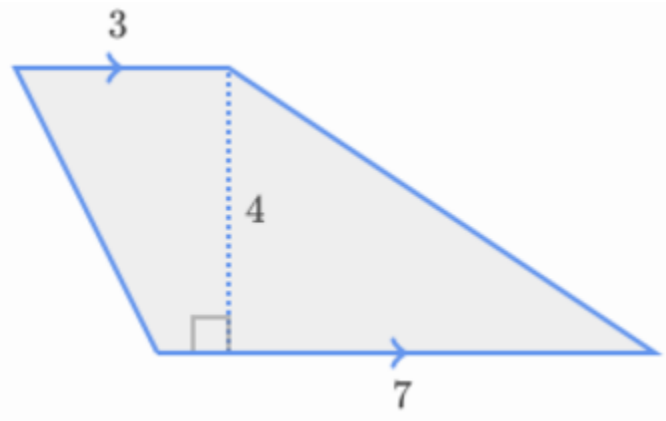


Problem 6: In rhombus ABCD to the right, find $\angle EBC$.



Trapezoid

- One pair of parallel sides
- $A = \bar{b}\,h$, where $\bar{b}$ = average of bases (and of course, h $\perp$ b!)



Problem 7: What is the area of this trapezoid?

Optimizing rectangles (sums, differences, products)

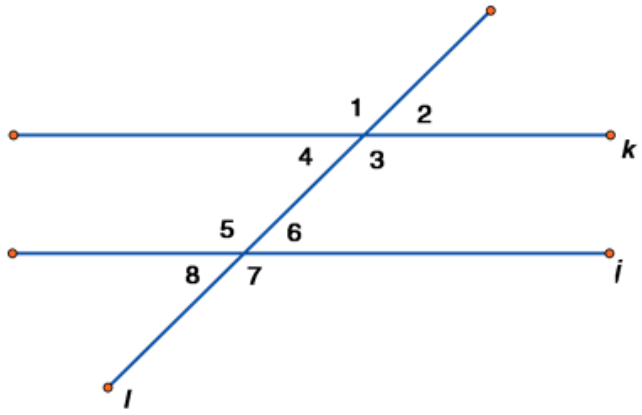
The more squarier, the more area.
The longer and skinnier, the more the perimeter.

Problem 8: A rancher purchased 4,000 yards of fence to build a rectangular cattle pen. What is the most land area that he can enclose with this fence?

Problem 9: Which two factors of 100 have the greatest sum?

Intersecting lines and angles

- Any two intersecting lines
  - Adjacent angles supplementary (add to 180)
  - "Opposite" (vertical) angles are equal
- Two parallel lines with transversal
  - The four acute angles are equal, and the four obtuse angles are equal
  - The acute and obtuse angles are supplementary
  - If the transversal is almost perpendicular, "slant" it for clarity
  - The same rules apply to parallel sides of a parallelogram / trapezoid



Problem 9: If $\angle 1 = 130^{\circ}$, then what is $\angle 7 - \angle 6$?

Three dimensions

Dimensionality

1D = "Linear". Length, width, height, etc. (measurable with a piece of string)
2D = Area / Planar (tiles)
3D = Volume / Spatial (water)

Every planar shape has 1D and 2D measurements! (Example?)
Every spatial shape has 1D, 2D, and 3D measurements! (Examples?)

When you sweep a shape perpendicular to itself, it creates a shape with higher dimensionality.
1D × 1D = 2D: Width x height = Area
1D × 2D = 3D: Area x height = Volume

Don't bother memorizing formulas. Just understand these principles.

Box shapes

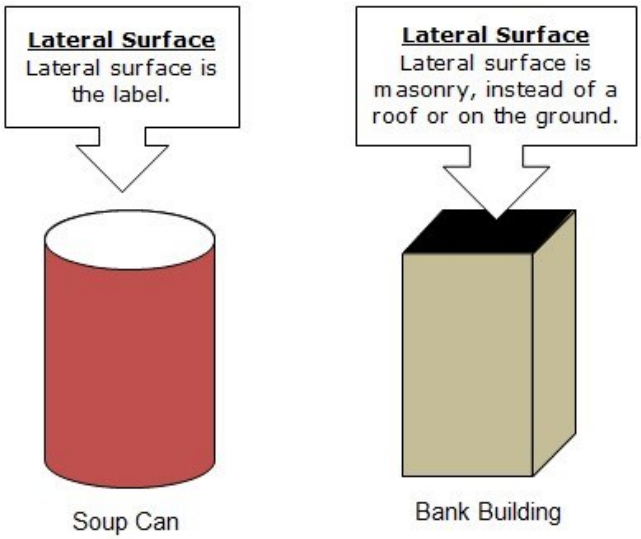
Base	× **height**	=
Perimeter / circumference (1D)	× height (1D)	= Lateral surface area (2D)
Area (2D)	× height (1D)	= Volume (3D)

The "height" must always, always, ALWAYS be perpendicular to the base!!!

Problem 11: The soup can has a radius of 5 cm and a height of 12 cm.

What is the area of its label?

How much metal is needed to make the can?



How much soup will the can hold?

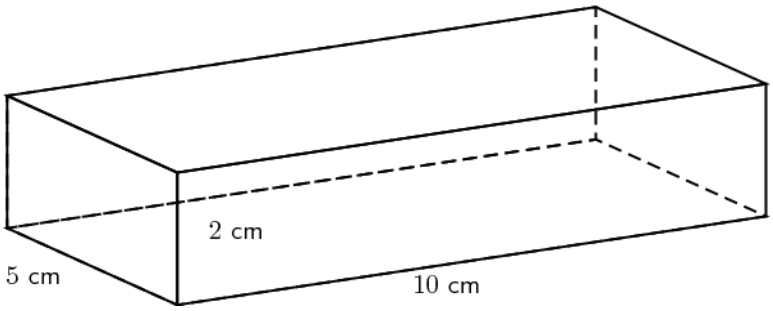
Not tested

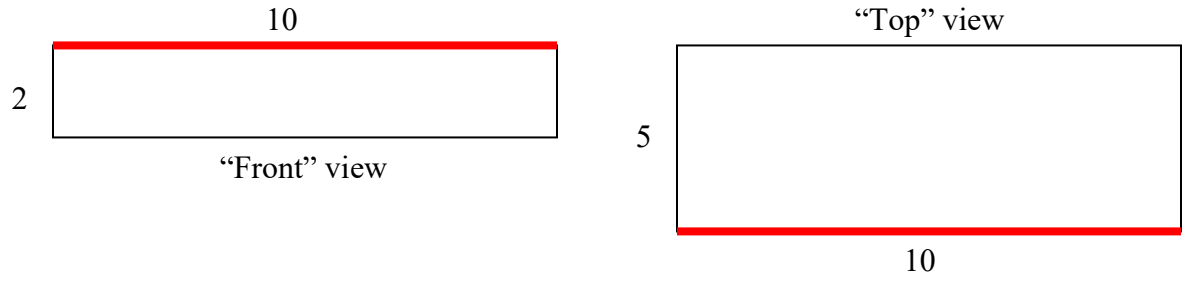
- Pyramids
- Cones
- Spheres
- Yaaayyyyyy!

Visualizing

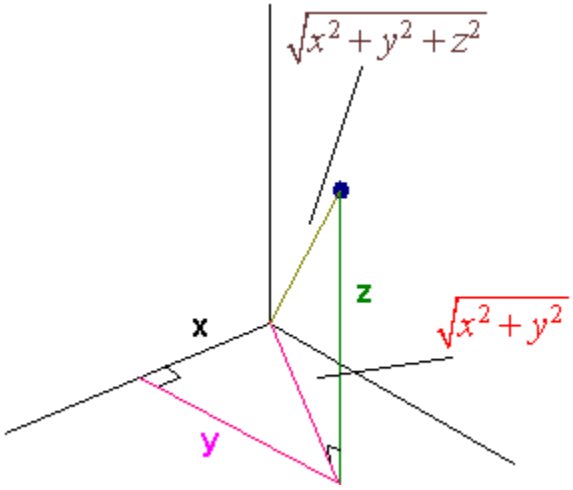
Draw shapes "straight on"
If shape is complex, draw from two or three directions
Identify measurements in common between views.

Problem 12: How much wrapping paper is required to wrap the small gift box below?





The 3D Pythagorean Theorem: "Diagonal of a box"



$$x^2 + y^2 + z^2 = d^2$$

Problem 13: Two mountain climbers are joined by 300 feet of rope. Mountain climber Abe is 100 feet to the north and 200 feet to the east of climber Ben. What is the maximum elevational difference between them?

Scaling

Similar figures: "Same shape, different size". Can be 1D, 2D, or 3D.

Let *l, a,* and *v* be a length, area, and volume of a small shape.

Let L, A, and V be the corresponding length, area, and volume of a larger similar shape.

Let r be the linear ratio comparing the two shapes:

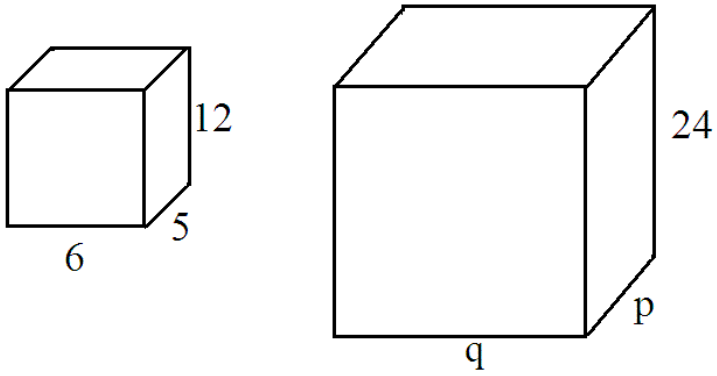
If $L = lr$, then

$A = ar^2$, and

$V = vr^3$

Regardless of the shape!

Problem 14: Find p and q given that these boxes are similar.



Problem 15: A dollhouse is an exact model of a real house with a scale factor of 20.

The real house has an 800 square foot garage. What is the area of the doll house's garage?

The doll house has a swimming pool that holds 0.5 gallons of water. How much water does the real swimming pool hold?