Algebra Power Hours 2 – 3

Systems of Equations

- Substitution
- Linear combinations

$$\begin{cases} 3m+5n=47 \\ n=2m-1 \end{cases} \qquad \begin{cases} ab=-24 \\ 2a-b=14 \end{cases}$$

$$\begin{cases} 4x+3y=23 \\ 2x-y=19 \end{cases} \qquad \begin{cases} 2x+5y=2 \\ 7x-3y=-34 \end{cases}$$

- An $m \times n$ system of equations (m equations in n unknowns) generally has $n - m$ degrees of freedom. You are free to choose values for $n - m$ of the variables, and then the rest will be determined.

$$\begin{cases} x+y+3z=30 \\ 5+4x+y=20 \end{cases}$$

A	B
x	3

A	B
x	z

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Rate Problems

A rate has fractional units like miles / hr. Therefore, it can be expressed in two reciprocal ways.

$$30\frac{mi}{hr} = \frac{30\ mi}{1\ hr} \quad = \quad \frac{1\ hr}{30\ mi} = \frac{1}{30}\frac{hr}{mi}$$

Shared Work

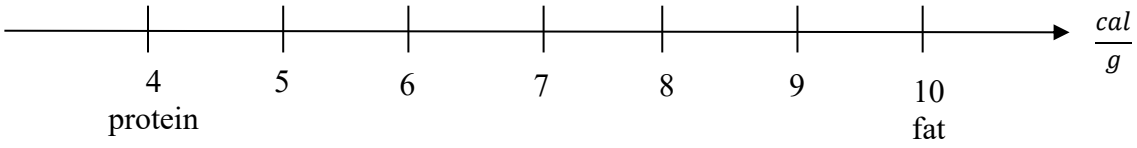
You can add / subtract rates of work, but only if units of time are in the denominator. Reciprocate whenever necessary! (See Ex. 2.7.7. on p. 246). How could you answer this problem by plugging in?

Fractional mixture problem / Weighted Average

You only need to understand a few simple principles of weighted averages in order to solve mixture problems on the GRE. We will illustrate them with this example.

Problem 10: Protein has 4 calories per gram, and fat has 10 calories per gram.
How many calories are in each gram of a protein milkshake?
Some answers will be ranges, some particular values.

a. The milkshake contains some protein and some fat
b. The milkshake contains equal amounts of protein and fat
c. The milkshake contains more protein than fat
d. Bonus round: The milkshake contains twice as much protein as fat



The principles we have learned:

1. Mixture must be intermediate between constituents
2. If constituents are mixed in equal amounts, mixture is at their midrange.
3. If there is more of one constituent than the other, mixture tips in that direction.
4. To calculate an actual weighted average, think of the mean as the "balancing point"

Average speed

Average speed = $\frac{Total\ distance}{Total\ time}$

Problem 11 A plane travels 3,000 miles across the US at 600 miles per hour. On the return trip, it travels 500 miles per hour. What is its average speed on the round trip? (It's NOT 550 mph!)

Special inequalities

Absolute values

When you see	Rewrite as	Solution
$5 < x < 10$	x is between 5 and 10, or $x > 5$ AND $x < 10$	Points between 5 and 10
$x < 5$ OR $x > 10$		Points outside 5 to 10
$\|x\| = 5$	$x = -5$ or $x = 5$	Two endpoints only
$\|x\| < 5$	$-5 < x < 5$	Between the endpoints
$\|x\| > 5$	$x < -5$ OR $x > 5$	Outside the endpoints (left OR right!)

Solve:

$$|3x - 2| \geq 12$$

$$|5x - 1| < 40$$

Quadratic

$\sqrt{x^2} = |x|$

Solve:

1. $x^2 = 36$
2. $x^2 < 49$

3. If n is an integer and $n^2 < 20$, how many possible values of n are there?

Reciprocals

Reciprocating each side of an inequality flips the inequality sign.

If $\frac{a}{b} < \frac{c}{d}$, then $\frac{b}{a} > \frac{d}{c}$ (as long as the fractions have the same sign)

Ex: x is a positive number. Solve for x.

$$\frac{2}{x} < \frac{8}{3}$$

Function Theory

Function notation and graphing

- The phrase "$y = f(x)$" means that y is a variable whose value depends on x.
- The phrase "$f(1) = 4$" means "When $x = 1$, $y = 4$," represented by the point (1, 4) on a graph.
- The phrase "$f(x) = 5x + 2$" means that whenever you choose a value of x, you have to plug it into the formula $5x + 2$ to obtain the corresponding y-value. If you do this for all possible values of x, you get a set of points that forms a certain curve.

Funny functions

A new function is defined for you, such as $x * y = 3x - 4y$. You can choose values for x and y and plug them in to get a numeric answer.

#x# = $x^2 - 1$. What is the value of #3# – #2#?

If x is even, @x = $3x - 3$; if x is odd, @x = $2x - 2$. What is the value of @(@(@5)?

1. 21
2. 40
3. 63
4. 117
5. 140

Telescoping Sequence

A special sequence where terms cancel each other out and the sequence "collapses" to a very short subsequence.

Ex: $a_n = n^2 - (n-1)^2$. Find the sum of the first 50 terms.

Solution:
$1^2 - 0^2 + 2^2 - 1^2 + 3^2 - 2^2 + \ldots + 50^2 - 49^2$
Everything cancels except $0^2 + 50^2 = 2{,}500$!