Arithmetic Power Hour(s)

The Ladder of Operations

Arithmetic is built on the six operations, which are all related to addition. Understanding their relationships will give us a unifying metaphor and visual for many concepts.

Multiplying and Dividing Issues

Integers

"Not fractions or decimals" = …, -3, -2, -1, 0, 1, 2, 3, …

When written as a variable, an ***in***teger is usually represented by a letter between i and n.

Divisors

A ***divisor*** (factor) of 12 = 12 *divided* by an integer if the answer is an integer

Always come in pairs

Searching for divisors: Start with 1*itself

Divisor pairs of 12 = **1***12

2*6 Put them in order: 1, 2, 3, 4, 6, 12

3*4

Problem 1:

(a) What are the *divisors* of 18?
(b) What are the *common divisors* of 12 and 18?
(c) What is the *greatest common divisor* of 12 and 18?

Prime numbers

A number is ***prime*** if it has exactly two divisors: 1 and itself.
A number that has more than two divisors is ***composite***.
1 is "neutral", neither prime nor composite.

1^{st} five prime numbers = 2, 3, 5, 7, 11

Every number has a unique ***prime factorization***, or product of prime numbers.
When you can't think of a number's divisor off the top of your head, try dividing by a small prime.

What is the prime factorization of 90?

Principle: Any combination of prime factors of a number is also a factor of that number.

Ex: 210 = $2 \cdot 3 \cdot 5 \cdot 7 = 2 \cdot 5 \cdot (3 \cdot 7)$ = $\mathbf{10} \cdot (3 \cdot 7)$, so 10 is a divisor (part) of 210.

<u>Problem 2</u>: A large number *M* has prime factorization $M = 2^5 \cdot 3 \cdot 5^3 \cdot 17$. Which number must be a divisor of *M*?

[A] 100 [B] 34 [C] 18

Multiples

Do not confuse multiples and divisors!

A multiple of 12 = 12 *times* an integer

List of multiples of 12: …, -36, -24, -12, 0, 12, 24, 36, …

= …, 12( ), 12( ), 12( ), 12( ), 12( ), 12( ), 12( ), …

Formula: 12*k*, where *k* is an integer

We usually use non-negative multiples: 0, 12, 24, 36, …

Common multiples

Common multiple problems involve repeated cycles (flashing lights, running laps, daily schedule, etc.)

<u>Problem 3</u>: The blue bus comes to this stop every 45 minutes and the green bus every 20 minutes. If they were both at this stop at 12:00 noon, at what times will they both be at this stop again?

Solution 1: List multiples of the large number (45) until you reach a multiple of the smaller number (20). 45, 90, 135, **180**

List the next three times that both buses will be at this stop again:

Solution 2: LCM(a, b) = $\frac{ab}{GCF(a,b)}$

To find LCM of several numbers, do them two at a time.

<u>Problem 4</u>: What if there's a yellow bus with lap time of 16 minutes?

Problems involving adding / subtracting *and* multiplying / dividing

Remainder notation

GRE language: "When n is divided by 7, the remainder is 3."
Translation: "n is (a multiple of 7) plus 3."
Formula: $n = 7k + 3$
List: 3, 10, 17, 24, …

Problem 5: When *m* is divided by 9, the remainder is 6. When *m* is divided by 5, the remainder is 2.

(a) What number must be a divisor of *m*?

(b) What is the smallest possible positive value of *m*?

Generic even / odd numbers

Question: Given some generic even / odd numbers, determine whether an expression is even or odd.
Method: Plug in 0 or 2 for the even numbers, and plug in 1 for the odd numbers.
Plugging in will always give you the correct answer as long as you are only adding, subtracting, multiplying, or raising to powers.

Problem 6: If *m* is even and *n* is odd, which number must be even?

[A] $3m + 2n$ [B] $m^2 - n$ [C] $m/2$

Evenly spaced numbers (Arithmetic sequence)

Recognizing an evenly spaced list of numbers

Consecutive integers, even integers, odd integers,
"Each term equals the preceding term plus a constant."

Principles

1. Number of terms in sequence: $n = \frac{range}{spacing} + 1$
2. Mean = Median = Midrange

Reminders!

Range =	Mean =
Median =	Midrange =

Problem 7: How many numbers are in the list 103, 107, 111, … , 199?

Problem 8: Add up all the integers from 1 to 99

Problem 9: Find 5 consecutive odd integers with a sum of 355

Exponent and Root Issues

The Rules of Exponents

Exponents operate "one level straight down" on the ladder of operations.

Problem 8: Simplify each expression using a single exponent.

(a) $x^2 \cdot x^3$ (b) $(x^2)^3$

(c) $\frac{x^{12}}{x^4}$ (d) $\sqrt[3]{x^{12}}$ (more on this below)

The Distributive / Factoring Properties

An operation can be distributed over another operation on the level below it.

Problem 9: True or False?

(a) $(x+3)^2 = x^2 + 9$ (b) $\sqrt{\frac{x^2}{16}} = \frac{x}{4}$ (c) $\frac{8x-4}{2} = 4x - 4$

Simplifying roots

The basic principle here is that an equal power and root cancel each other: $\sqrt[4]{x^4} = x$, $\sqrt{5^2} = 5$.

- The exponent should be less than the root
- The whole number in the radicand should be as small as possible.

	Factoring out a perfect square	Factoring out a perfect cube
Numeric example	$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$	$\sqrt[3]{24} = \sqrt[3]{8 \cdot 3} = \sqrt[3]{8} \cdot \sqrt[3]{3} = 2\sqrt[3]{3}$
Variable example	$\sqrt{x^5} = \sqrt{x^4 \cdot x} = \sqrt{x^4} \cdot \sqrt{x} = x^2\sqrt{x}$	$\sqrt[3]{x^{14}} = \sqrt[3]{x^{12} \cdot x^2} = \sqrt[3]{x^{12}} \cdot \sqrt[3]{x^2} = x^4\sqrt[3]{x^2}$