

Proportions and Percents 'til you Puke

Arithmetic

I think I'm gonna be sick ...

Fraction / Decimal / Percent Conversions

D \leftrightarrow P: Shift dec. pt. 2 places (multiply or divide by 100%)

1) $62.5\% =$ _____ (decimal)

2) $0.8319 =$ _____ %



P \rightarrow F: Percent means "over 100": $41\% =$ _____ (fraction)

F \rightarrow P: The hardest; usually requires decimal intermediary.

$\frac{3}{8} =$ _____ (decimal) = _____ %

Percent of a number

Percents are *relative* numbers, incomplete without a base.

Ex. 30% of 250 (250 is the base)

"of" means "times"

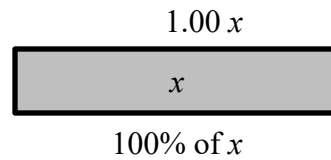
$$30\% \text{ of } 250 = 0.3 \times 250$$

3) Cumulative percent: 20% of 85% of 140% of 1,200

$$= \text{_____} \times \text{_____} \times \text{_____} \times \text{_____}$$

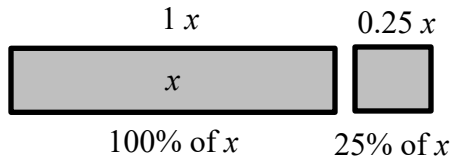
Percent more / less than

Every number is 1 of itself.
Every number is 100% of itself.

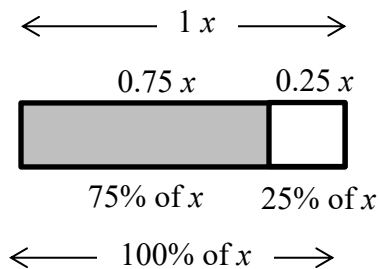


Therefore, percent changes (more / less than) are always derived from 100%.

$$25\% \text{ more than } x = 125\% \text{ of } x = 1.25 x$$



$$25\% \text{ less than } x = 75\% \text{ of } x = 0.75 x$$



- 4) 32% more than $x = \underline{\hspace{1cm}}$ % of $x = \underline{\hspace{1cm}}$ x
- 5) 15% less than $x = \underline{\hspace{1cm}}$ % of $x = \underline{\hspace{1cm}}$ x
- 6) $1.14x = \underline{\hspace{1cm}}$ % of $x = \underline{\hspace{1cm}}$ % more / less than x
- 7) $0.7x = \underline{\hspace{1cm}}$ % of $x = \underline{\hspace{1cm}}$ % more / less than x
- 8) 200% more than $x = \underline{\hspace{1cm}}$ % of $x = \underline{\hspace{1cm}}$ x
- 9) $7x = \underline{\hspace{1cm}}$ % of $x = \underline{\hspace{1cm}}$ % more than x

Misconception # 1: Cumulative percent changes

Student: "Percent changes accumulate by adding / subtracting. If a price goes up by 25% and then down by 5%, altogether it has gone up by 20%."

WRONG 😞

Why: You can't add / subtract percents of different bases.

10) So what is 5% less than 25% more than P ?

= _____ % of _____ % of P

= (.) (.) P (fill in decimals)

= (.) P (decimal)

= _____ % of P

= _____ % more than P

11) A car was listed on eBay for \$6,000. When it didn't sell, Selma the Seller reduced the price by 10%. Then Bart the Buyer bought it. Bart had to pay a 3% eBay fee. What was Bart's total expense?

Misconception # 2: Interchanging bases

Student: "If the teacher is 25% older than me, then I'm 25% younger than the teacher."

WRONG 😞

Solution: Interchanged percents are reciprocals of each other.

12) If the teacher is 25% older than the student, then the student is what % younger than the teacher?

Reword into standard math-English:

The teacher's age (T) is 25 % more than the student's age (S)

T is _____ % of S

T is _____ of S (fraction)

S is _____ of T (fraction) Reciprocate.

Now reverse the steps, but we need a decimal to get from fraction to percent:

S is 0._____ of T (decimal)

S is _____ % of T

S is _____ % less than T

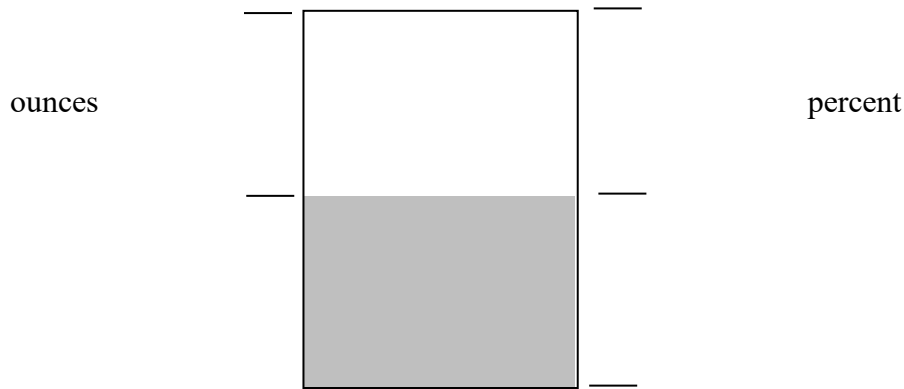
Algebra

Proportion: Cross-multiply

$$13) \frac{3}{5} = \frac{27}{x}$$

$$14) \frac{x+2}{3} = \frac{2x-1}{4}$$

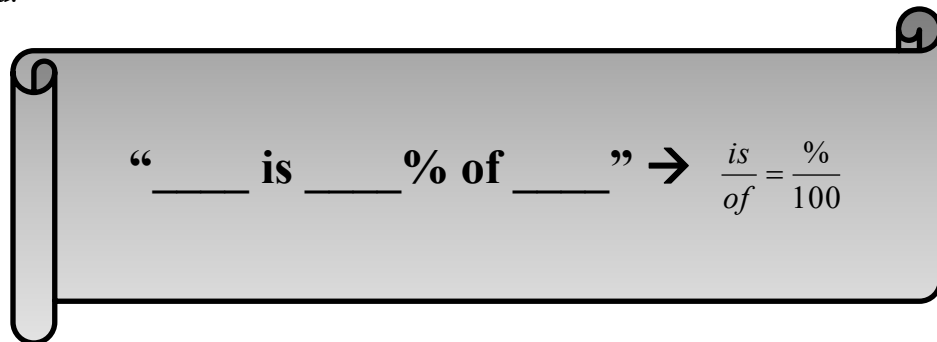
The percent-o-meter. Label a half-full, 12 ounce can with ounces and percents.



Percent-as-proportion

“ ___ is ___ % of ___ ” becomes $\frac{6}{12} = \frac{50}{100}$

Generalized:



15) 18 is 30% of what number?

Translating “Word Problems”

Almost all percent problems model one of the following abstract prototypes:

1. Comparison is some % of base.

Pure English example: John only earns 88% as much as Mary.

Translation: _____ is 88 % of _____.

2. Part is some % of whole

Pure English example: What % of the cookies are chocolate?

Translation: _____ is what % of _____ ?

3. New is some % of old

Pure English example: Compared to a year ago, the stock has retained only 75% of its value.

Translation: _____ is 75 % of _____ .

Students get percent problems wrong when they don't know *exactly* which numbers go where. The solution to this uncertainty is to take *several small translation steps*, rather than trying to leap to the solution in one giant bound!!!

- 16) A team has 16 men and 24 women. What percent of the team is men?

Reword as one of the three prototypes: Part is some % of whole

Restate with problem-specific terms:

_____ is _____ % of _____.

Fill in numbers or letters: _____ is _____ % of _____

Percent change or difference problems

All the prototypes above can be reformulated as a change or difference question:

_____ is _____ % more / less than _____

We must reword “more / less than” to “of” before using proportions!!!

If the percent is known, this is easy.

17) A product is on sale at 40% off regular price, for \$360. What was the regular price?

Prototype: _____ is some % more / less than _____

Specific terms: _____ is some % less than _____

Plug in numbers or x : _____ is _____ % less than _____*

Reword using “of”: _____ is _____ % of _____

Solution: \$600

If the percent is unknown: Subtract and keep the base.

Example: 16 is what percent less than 20?

Subtract and keep the base: 4 is x % of 20

18) The population of a city increased from 80,000 in 2000 to 100,000 in 2020. By what percent did it increase?

Prototype: _____ is some % more than _____

Using problem-specific terms:

_____ is what % more than _____?

Plug in numbers: _____ is _____ % more / less than _____

Subtract and keep the base: _____ is x % of _____

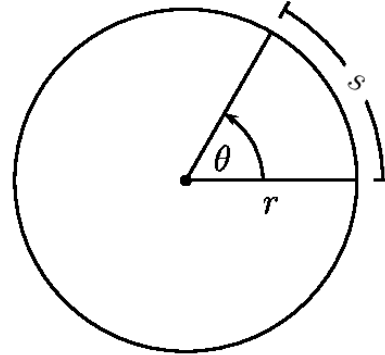
* Suppose your friend wrote “ R is 40% more than 360.” Would this be correct?

Geometry / Data

Circles / pie charts

Reminder formulae

- $d = 2r$
- $c = d\pi$. A partial circumference is an arclength, s .
- $A = r^2\pi$
- 360 degrees in a full circle.
- Practice: If $r = 6$, find d , c , and A .



The “Slice of Pie” Formula

“Just about everything” on a circle or pie chart is proportional to the central angle.

$$\frac{\text{arclength}}{\text{circumference}} = \frac{\text{Sector area}}{\text{Whole area}} = \frac{\text{Partial number}}{\text{Whole number}} = \frac{\%}{100} = \frac{\theta}{360}$$

Which can be summarized and memorized as

$$\frac{\text{Slice}}{\text{Pie}} = \frac{\theta}{360}$$

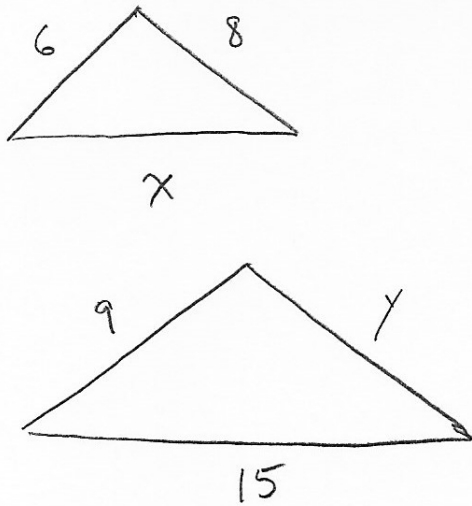
19) If a circle has a radius of 6, find the arclength of a sector with central angle 80 degrees.

20) 108 out of 540 vacationers are going to the beach for Memorial Weekend. On a pie chart, how large should the central angle be for this category of vacationers?

Similar triangles

- Angles are all equal
- Have proportional sides
- A parallel cut makes similar triangles: Helpful to draw them separately

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