



nth Degree Tutoring

Math | Science | English | SAT | ACT | GRE | LSAT | Admissions

Probability

Mix-and-match

_____ Basic probability
_____ AND
_____ OR
_____ NOT

A. Addition
B. Subtraction
C. Multiplication
D. Division

Basic problems

Question: If you select a choice, what is the probability that it will be a success?

Answer: $\frac{n(\text{Successes Within the Choices})}{n(\text{Choices})}$

Empirical probability table: Make row / column totals!

The following table shows the number of website subscribers who chose various kinds of subscription options.

	Basic membership	Premium membership	Gold membership
Monthly	8,000	6,000	2,000
Annual	5,000	2,000	1,000

If you choose a subscriber, what is the probability she chose a premium membership?

If you choose a gold member, what is the probability that he subscribes monthly?

If you choose a monthly member, what is the probability that she has a gold membership?

AND, OR, and NOT.

	Basic membership	Premium membership	Gold membership	Total
Monthly	8,000	6,000	2,000	16,000
Annual	5,000	2,000	1,000	8,000
Total	13,000	8,000	3,000	24,000

If you choose a monthly member, what is the probability that he is NOT a gold member?

If you choose a subscriber, what is the probability that he is a basic member AND is an annual subscriber? (AND = “overlap”) Note that this could easily be worded without the word “AND”; the translation is vital.

If you choose a subscriber, what is the probability that she is a basic member OR a premium member? (OR = “all-encompassing”)

$$P(A \text{ OR } B) = P(A) + P(B)$$

Must subtract overlap:

If you choose a subscriber, what is the probability that she is a basic member OR a monthly subscriber?

$$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$$

Multiplication:

Draw a card and roll a die. What is the probability that both are a 5?

Reword: The die is a 5 AND the card is a 5.

Multiply:

Adjust if the first selection affects the second one (“conditional” probability)

Draw two cards from a deck. What is the probability that both are 5's?

Counting Principles

Overlapping sets

300 diners went through a Thanksgiving buffet with a choice of ham and / or turkey.
 100 had ham.
 150 had turkey.
 20 had both ham and turkey.
 How many diners had exactly one kind of meat?

	Ham	No ham	Total
Turkey			
No turkey			
Total			

300 diners went through a Thanksgiving buffet with a choice of ham and / or turkey.
 100 had ham.
 150 had turkey.
 What is the least / greatest amount of diners who could have had no meat?

	Ham	No ham	Total
Turkey			
No turkey			
Total			

Multiplication / Powers:

How many CA license plates can be made of the form #AAA### ?

What if no repetitions are allowed?

Combinatorics

Multiplying descending integers is the basic counting principle behind “combinatorics”, or counting the number of ways to make selections without repetition.

First, recap difference between “set” and “sequence”.

Set = The grocery bag with all your grocery items lumped inside.

Sequence = The individual grocery items lined up on the checkout line.

A “sequence” can be indicated by any situation involving order, rank, labels, or distinctions of any kind. (Different orderings = different sequences)

One set can be arranged into many sequences.

Factorial

Orderings of an entire set into a sequence.

If your grocery basket has six items in it, in how many different orders can they be rung up on the checkout line?

Memorize factorials up to $6! = 720$

$1! = 1$	$2! = 2 \times 1 = 2$	$3! = 3 \times 2 \times 1 = 6$
$4! = 4 \times 3 \times 2 \times 1 = 24$	$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$	$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Permutation

Orderings of part of a set (a subset) into a sequence.

If your grocery basket has six items in it but you can only afford four of them, in how many orders can you ring up four of them on the checkout line?

$${}_6P_4 = \underline{\hspace{1cm}} \underline{\hspace{1cm}} \underline{\hspace{1cm}} \underline{\hspace{1cm}} =$$

Combination

Selections of subsets (no order, rank, distinction, etc)

If your grocery basket has eight items in it but it's too heavy and you have to remove three of them into a different bag, how many combinations of items can be in that bag?

$${}_8C_3 = \frac{{}_8P_3}{3!} = \frac{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}} =$$

What do you notice about the number of combinations left for the original bag? What does this tell you about ${}_8C_3$ and ${}_8C_5$?